MATHEMATICS (ELECTIVE) 2

1. GENERAL COMMENTS

The Chief Examiners for Mathematics (Elective) reported that the standard of the paper compared favorably with that of the previous year.

2. CANDIDATES' PERFORMANCE

The Chief Examiners for Mathematics (Elective) reported that the performance was better than that of the previous year.

3. SUMMARY OF CANDIDATES' STRENGTHS

Candidates' strengths were evident in the following areas:

(a) finding common ratio, nth term of a Geometric Progression;
(b) finding gradient of a curve at a point;
(c) solving probability related problems;
(d) resolving forces into component form;
(e) finding inverse and partial fraction.

4. SUMMARY OF CANDIDATES' WEAKNESSES

Candidates showed weaknesses in the following:

(a) writing required angles in trigonometric equation in a given range;
(b) applying approximations in solving questions;
(c) wrong application of calculator instead of showing workings;
(d) use of wrong formula in solving questions;
(e) sketching a force diagram;
(e) simplifying expression involving combination.

5. SUGGESTED REMEDIES FOR THE WEAKNESSES

a) Teachers should prepare the candidates very well by giving them more exercise particularly on the above weaknesses and guide them to work.
b) Candidates should be encouraged to solve more problems relating to these areas identified so that they can understand the underlying concepts.

6. DETAILED COMMENTS

Question 1

Given the exponential sequence (*G. P*) $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots$, find the;

(a) **Common ration;**

(b) **9^{th} term;**

(c) **Sum of the first nine terms, of the sequence.**

This question was very popular and well answered by candidates. However, in finding the sum of the first 9 terms, some candidates failed to simplify after the substitution. They use calculator to arrive at the answer which makes it not judicious. Candidates were expected to answer the question as follows:

(a) $\quad$ common ratio $= \frac{1}{4} \div \frac{1}{2}$

$$= \frac{1}{2}$$

(b) $\quad 9^{\text{th}}$ term $= \frac{1}{2}\left(\frac{1}{2}\right)^8$

$$= \left(\frac{1}{2}\right)^9$$

$$= \frac{1}{512} \text{ or } 0.00195$$

(c) $\quad S_9 = \dfrac{\frac{1}{2}\left(1-\left(\frac{1}{2}\right)^9\right)}{1-\frac{1}{2}}$

$$= 1 - \frac{1}{512}$$

$$= \frac{511}{512} \text{ or } 0.998$$

Question 2

Solve $4\tan\boldsymbol{\theta} = 3$, $0° \leq \boldsymbol{\theta} \leq 360°$, correct to the nearest degree.

Most candidates use the identities to simplify and solve the equation well. However, some candidates obtained two (2) angles instead of four (4). Some also did not correct the angles to the nearest degree. Candidates were expected to answer the question as follows:

$$4sin2\theta tan\theta = 3$$

$$4(2sin\theta cos\theta)\left(\frac{sin\theta}{cos\theta}\right) = 3$$

$$8sin^2\theta = 3$$

$$sin^2\theta = \frac{3}{8}$$

$$sin^2\theta = 0.375$$

$$sin\theta = \pm 0.6124$$

$$\theta = sin^{-1}(\pm 0.6124)$$

$$= \pm 37.76°$$

$$\theta = 180° - 37.76°$$

$$= 142.24°$$

$$\theta = 180° + 37.76°$$

$$= 217.76°$$

$$\theta = 360° - 37.76°$$

$$= 322.24°$$

$$\theta = (38°, 142°, 218°, 322°)$$

Question 3

Using trapezoidal rule with nine ordinates, find, correct to two decimal places, the area of the region bounded by the curve $y = \frac{1}{1+x^2}$, the x-axis and the lines $x = 0$ and $x = 8$.

Candidates constructed the table and use the rule to solve the question very well. Few candidates left the table in 2 decimal place and also use wrong rule. Candidates were expected to answer the question as follows:

$$h = 1$$

x	0	1	2	3	4	5	6	7	8
$\frac{1}{1+x^2}$	1	0.5	0.2	0.1	0.0588	0.0385	0.0270	0.02	0.0154

$$\int_0^8 \frac{1}{1+x^2}\,dx = \frac{1}{2}(1)[1 + 0.0154 + 2(0.5 + 0.2 + 0.1 + 0.0588 + 0.0385 + 0.0270 + 0.02)]$$

$$= 0.5(1.0154 + 1.8886)$$

$$= 0.5(2.9040)$$

$$= 1.452$$

$$= 1.45$$

Question 4

Find the equation of the normal to the curve $y = \frac{2x+1}{x-1}, x \neq 1$ at the point (0, -1).

The candidates differentiated the curve, put the point and found the gradient of the tangent and also the gradient of normal. They went on to find the equation. The question was well answered by candidates. Candidates were expected to solve the question as follows:

$$y = \frac{2x+1}{x-1}$$

$$\frac{dy}{dx} = \frac{2(x-1) - 1(2x+1)}{(x-1)^2}$$

$$\frac{dy}{dx}_{(0,-1)} = \frac{2(0-1) - 1(2(0)+1)}{(0-1)^2}$$

$$= \frac{-2-1}{1}$$

$$= -3$$

$$\text{Gradient of normal} = \frac{1}{3}$$

$$y + 1 = \frac{1}{3}(x - 0)$$

$$3y - x + 3 = 0$$

Question 5

The ages (in years) of 9 members of a social club are 17, 11, 31, 18, 25, 12, 29, 13 and 15. Calculate the mean deviation.

Candidates were given a data to calculate mean deviation. This was answered well by most of the candidates. However, few did not calculate the mean before the mean deviation there by making it not judicious. Candidates were expected to solve the question as follows:

$$Mean = \frac{171}{9}$$
$$= 19$$

x	$x-\bar{x}$	$\lvert x-\bar{x}\rvert$
11	-8	8
17	-2	2
31	12	12
18	-1	1
25	6	6
12	-7	7
29	10	10
13	-6	6
15	-4	4
		$\sum\lvert x-\bar{x}\rvert = 56$

$$Mean\ Deviation = \frac{56}{9}$$
$$= 6.2222$$

Question 6

A group consists of 8 men and 7 women. If two representatives are to be selected from the group at random without replacement for a debate competition, find the probability that they are:

(a) **Both women;**

(b) **Of the same sex.**

Probability of both women were answered well by candidates. However, in the probability of same sex, most of the candidates used the calculator without simplifying to get the answer. Candidates were expected to solve the question as follows:

(a) $$P(both\ women) = \frac{\binom{7}{2}}{\binom{15}{2}}$$
$$= \frac{21}{105}$$

$$= \frac{1}{5} \text{ or } 0.2$$

$$\text{(a)} \quad P(the\ same\ sex) = \frac{\binom{8}{2}}{\binom{15}{2}} + \frac{\binom{7}{2}}{\binom{15}{2}}$$

$$= \frac{28}{105} + \frac{21}{105}$$

$$= \frac{49}{105}$$

$$= \frac{7}{15} \text{ or } 0.4667$$

Question 7

The position vectors of A and B relative to an origin are given as $\overline{OA} = 2\boldsymbol{i} - 8\boldsymbol{j}$ and $\overline{OB} = 5\boldsymbol{i} + \boldsymbol{j}$. Find $\angle AOB$ and state the type of angle it is.

The candidates calculated the dot product, and length of the vectors. The question was well answered by candidates. Candidates were expected to solve the question as follows;

$$\overrightarrow{OA} \cdot \overrightarrow{OB} = \binom{2}{-8} \cdot \binom{5}{1}$$

$$= 10 - 8$$

$$= 2$$

$$cos\theta = \frac{2}{(\sqrt{4+64})(\sqrt{25+1})}$$

$$cos\theta = \frac{2}{(\sqrt{68})(\sqrt{26})}$$

$$\theta = cos^{-1}\left(\frac{2}{(\sqrt{68})(\sqrt{26})}\right)$$

$$= 87.27°$$

$\therefore \angle AOB$ is an acute angle

Question 8

A bullet of mass 25 *g* is fired with velocity (80*i* – 200*j*) *ms*$^{-1}$ into a wooden block of mass 4 *kg* which is at rest. If the bullet got stuck in the block, find the common velocity of the bullet and the block.

Candidates were to find common velocity of a bullet fired into a wooden block. This question was well answered by candidates. Meanwhile, few candidates failed to convert the masses to a common unit. They solved the question with different units of the masses. Candidates were expected to answer the question as follows:

$$m_1 = 25\ g$$
$$= 0.025\ kg$$
$$0.025(80\boldsymbol{i} - 200\boldsymbol{j}) + 4(0) = (0.025 + 4)v$$
$$2\boldsymbol{i} - 5\boldsymbol{j} = 4.025v$$
$$v = \frac{80}{161}\boldsymbol{i} - \frac{200}{161}\boldsymbol{j} \text{ or } 0.4969\boldsymbol{i} - 1.2422\boldsymbol{j}$$

Question 9

(a) **Express** $\frac{2x + 1}{x^2 - 8x + 16}$ **in partial fractions.**

(b) **If** $^{8}C_n : {}^{6}C_{(n-1)} = 56 : 15$**, find the values of** n**.**

This was a popular question attempted by candidates. In the part (a), candidates expressed the function into partial fraction and solve for the constants very well.

In the part (b), most candidates applied the combination and ratio very well but could not simplify to find the value of *n* in the question. Candidates were expected to answer the question as follows:

(a) $$\frac{2x+1}{(x-4)^2} \equiv \frac{A}{(x-4)} + \frac{B}{(x-4)^2}$$
$$2x + 1 = A(x-4) + B$$
$$2x + 1 = Ax - 4A + B$$
Comparing coefficients
$$A = 2$$
$$B - 4A = 1$$
$$B - 4(2) = 1$$
$$B - 8 = 1$$
$$B = 9$$
$$\frac{2x+1}{x^2 - 8x + 16} \equiv \frac{2}{(x-4)} + \frac{9}{(x-4)^2}$$

(b) $$\frac{8!}{n!(8-n)!} : \frac{6!}{(n-1)!(7-n)!} = 56 : 15$$
$$\frac{8!}{n!(8-n)!} \times \frac{(n-1)!\,(7-n)!}{6!} = \frac{56}{15}$$
$$\frac{56}{n(8-n)} = \frac{56}{15}$$
$$(8n - n^2)56 = 56 \times 15$$
$$n^2 - 8n + 15 = 0$$
$$(n-5)(n-3) = 0$$
$$n = 3 \text{ or } n = 5$$

Question 10

(a) A function g is defined as $g : x \to \frac{1}{2} - \frac{3x}{1 - x}$, Find the image of 1 under g^{-1} (the inverse of g).

(b) A curve has an equation $y = \frac{x}{2 + x^2}$. Find, correct to one decimal place, the gradient of the curve at $x = \frac{1}{2}$.

In the part (a), candidates were to find the image of 1 under the invers of g. Most interchanged x and y and make y the subject. They however failed to write the g^{-1} before finding the image of 1. Few who wrote g^{-1} failed to impose the domain to the g^{-1}.

In the part (b), Candidates were to find gradient of a curve at $x = \frac{1}{2}$. Most candidates differentiated the curve well and substituted candidates who attempted this question used calculator to simplify to the gradient. It was not accepted. Candidates were expected to solve the question as follows;

(a) $g(x) = \frac{1}{2} - \frac{3x}{1-x}$

$$= \frac{1 - x - 2(3x)}{2(1-x)}$$

$$y = \frac{1-7x}{2-2x}$$

$$x = \frac{1-7y}{2-2y}$$

$$x(2-2y) = 1 - 7y$$

$$y(7-2x) = 1 - 2x$$

$$y = \frac{1-2x}{7-2x}$$

$$g^{-1}(x) = \frac{1-2x}{7-2x}, \quad x \neq \frac{7}{2}$$

$$g^{-1}(1) = \frac{1-2(1)}{7-2(1)}$$

$$= -\frac{1}{5}$$

(b) $\frac{dy}{dx} = \frac{(2+x^2)(1) - x(2x)}{(2+x^2)^2}$

$$= \frac{2 - x^2}{(2+x^2)^2}$$

$$\frac{dy}{dx}_{(x=\frac{1}{2})} = \frac{2 - \left(\frac{1}{2}\right)^2}{\left(2 + \left(\frac{1}{2}\right)^2\right)^2}$$

$$= \frac{\frac{7}{4}}{\left(\frac{9}{4}\right)^2}$$

$$= \frac{7}{4} \times \frac{16}{81}$$

$$= 0.3457$$

$$= 0.3$$

<u>Question 11</u>

(a) **Two linear transformation, P and Q are defined by**

$\boldsymbol{P : (x, y) \rightarrow (2x + y, x + 2y)}$

$\boldsymbol{Q : (x, y) \rightarrow (2x + 3y, 2y)}$

(i) Write down the matrices of P and Q.

(ii) Find the image of (2, -4) under the transformation P followed by Q.

(b) **Solve $\log x = 1 + 2\log y$ and $2x = 9y - 1$ simultaneously.**

In the part (a), candidates were to find the image of P followed by Q. An average number of candidates who attempted this question quoted matrix Q wrongly. Instead of $Q = \begin{pmatrix} 2 & 3 \\ 0 & 2 \end{pmatrix}$, they got $Q = \begin{pmatrix} 2 & 3 \\ 0 & 1 \end{pmatrix}$. The concept of finding the image was well done but almost all the candidates who attempted left the image in column form instead of coordinates form.

In the part (b), candidates removed the log and solved the simultaneous equation well. Candidates were expected to solve the question as follows:

(a) (i) $P = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$

$Q = \begin{pmatrix} 2 & 3 \\ 0 & 2 \end{pmatrix}$

(ii) $QP = \begin{pmatrix} 2 & 3 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$

$= \begin{pmatrix} 7 & 8 \\ 2 & 4 \end{pmatrix}$

$= \begin{pmatrix} 7 & 8 \\ 2 & 4 \end{pmatrix}\begin{pmatrix} 2 \\ -4 \end{pmatrix}$

$= \begin{pmatrix} -18 \\ -12 \end{pmatrix}$

$\therefore$ The image is $(-18, -12)$

(b) $logx = 1 + 2logy$

$logx = log10 + 2logy$

$x = 10y^2$ …….(1)

$2x = 9y - 1$…..(2)

Substituting (1) into (2)

$2(10y^2) = 9y - 1$

$20y^2 - 9y + 1 = 0$

$(4y - 1)(5y - 1) = 0$

$$y = \frac{1}{4} \text{ or } y = \frac{1}{5}$$

$$when\ y = \frac{1}{4}$$

$$x = 10\left(\frac{1}{4}\right)^2$$

$$x = \frac{5}{8}$$

$$when\ y = \frac{1}{5}$$

$$x = 10\left(\frac{1}{5}\right)^2$$

$$x = \frac{2}{5}$$

<u>**Question 12**</u>

Fifteen percent of the participants in a conference presented papers. If a random sample of 10 participants is taken, find the probability that:

(a) **exactly three;**

(b) **not more than two;**

(c) **not less than eight;**

presented paper.

Candidates were to use binomial probability to find probability that exactly three, not more than two and not less than eight presented papers.

This question was answered well by candidates. However, few still used calculator where the question demand simplification. On a whole it was well answered by most of the candidates who attempted. Candidates were expected to solve the question as follows:

(a) $p = 15\% = \frac{3}{20}, q = \frac{17}{20}, n = 10$

$$P(at\ exactly\ 3) = P(x = 3)$$

$$P(x = 3) = \binom{10}{3}\left(\frac{3}{20}\right)^3\left(\frac{17}{20}\right)^7$$

$$= 120 \times \frac{27}{8000} \times \left(\frac{17}{20}\right)^7$$

$$= 0.12983$$

$$= 0.1298$$

(b) $P(\text{not more than } 2) = P(x \leq 2)$

$$P(x \leq 2) = P(x = 0) + P(x = 1) + P(x = 2)$$

$$P(x \leq 2) = \binom{10}{0}\left(\frac{3}{20}\right)^0\left(\frac{17}{20}\right)^{10} + \binom{10}{1}\left(\frac{3}{20}\right)^1\left(\frac{17}{20}\right)^9 + \binom{10}{2}\left(\frac{3}{20}\right)^2\left(\frac{17}{20}\right)^8$$

$$= 0.196874 + 0.347425 + 0.275896$$

$$= 0.820195$$

$$= 0.8202$$

(c) $P(\text{not less than } 8) = P(x \geq 8)$

$$P(x \geq 8) = P(x = 8) + P(x = 9) + P(x = 10)$$

$$P(x \geq 8) = \binom{10}{8}\left(\frac{3}{20}\right)^8\left(\frac{17}{20}\right)^2 + \binom{10}{9}\left(\frac{3}{20}\right)^9\left(\frac{17}{20}\right)^1 + \binom{10}{10}\left(\frac{3}{20}\right)^{10}\left(\frac{17}{20}\right)^0$$

$$= 0.0000083325 + 0.000000326768 + 0.0000000057665$$

$$= 0.000008665$$

Question 13

(a) The numbers 1, 2, 3, 4 and 5 have frequencies *p*, 11, *q*, 8 and 9 respectively. If the total frequency is 50 and the arithmetic mean is 2.7, find the values of *p* and *q*.

(b) The Spearman's rank correlation coefficient of 20 students marks in two tests is 0.8562. Find Σd^2 (sum of squares of the rank difference for each pair of ranks).

This was unpopular question. Few candidates who attempted were to find the values of p and q and concept of mean to formal two (2) equations and solve for p and q very well. Only few used calculators to the p and q simultaneously.

In the part (b), candidates were to find Σd^2 in the spearman's rank correlation coefficient. Out of few who attempted, majority quoted the formula wrongly. Candidates were expected to solve the question as follows:

(a)

x	f	fx
1	p	p
2	11	22
3	q	$3q$
4	8	32
5	9	45
	$\sum f = p + q + 28$	$\sum fx = p + 3q + 99$

$$p + q + 28 = 50$$

$$p + q = 22 \text{ ....(i)}$$

$$\frac{p + 3q + 99}{50} = 2.7$$

$$p + 3q + 99 = 135$$

$$p + 3q = 36 \text{ ......(ii)}$$

$$2q = 14$$

$$q = 7$$

$$p + 7 = 22$$

$$p = 15$$

(a) $0.8562 = 1 - \frac{6\sum d^2}{20(20^2 - 1)}$

$$\frac{6\sum d^2}{20(400 - 1)} = 1 - 0.8562$$

$$\frac{6\sum d^2}{20 \times 399} = 0.1438$$

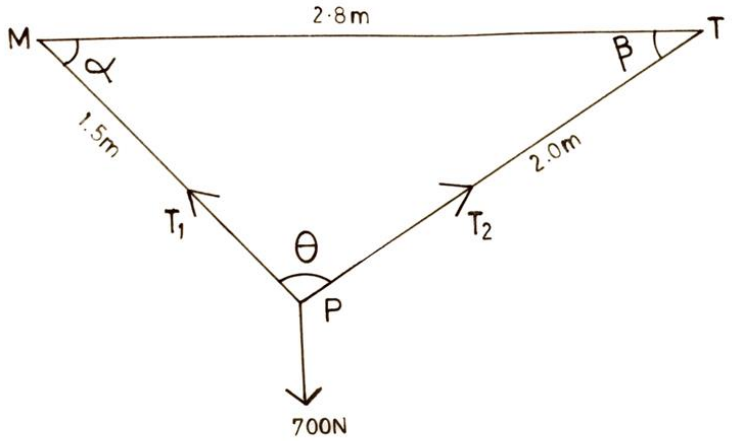
$$\sum d^2 = \frac{1147.524}{6}$$

$$\sum d^2 = 191.254$$

<u>Question 14</u>

A body weighing 700 N is suspended at point P by two inextensible strings at points M and T, where $|PM| = 1.5\ m, |PT| = 2.0\ m$ and $|MT| = 2.8\ m$. If the body remains in equilibrium, calculate to three significant figures, the tension in each string.

This question was not popular among candidates. Candidates were to calculate the tension in strings given the length of strings and distances between the point *M* and *T*. Few candidates who attempted could not sketch the force diagram was not done well. Few who were able to solve failed to correct the tensions in the string to three significant figures as the question demanded. Candidates were expected to solve the question as follows:



$$\cos\theta = \frac{2^2 + 1.5^2 - 2.8^2}{2 \times 1.5 \times 2}$$

$$= -\frac{1.59}{6}$$

$$= -0.265$$

$$\theta = \cos^{-1}(-0.265)$$

$$= 105.4°$$

$$\frac{2.8}{\sin 105.4°} = \frac{2}{\sin\alpha} = \frac{1.5}{\sin\beta}$$

$$\sin\alpha = \frac{2\sin 105.4°}{2.8}$$

$$= 0.6886$$

$$\alpha = \sin^{-1}(0.6886)$$

$$= 43.5°$$

$$\sin\beta = \frac{1.5 \times \sin 105.4°}{2.8}$$

$$= 0.5165$$

$$\beta = \sin^{-1}(0.5165)$$

$$= 31.1°$$

$$\frac{700}{\sin 105.4°} = \frac{T_1}{\sin 121.1°} = \frac{T_2}{\sin 133.5°}$$

$$T_1 = \frac{700 \times \sin 121.1°}{\sin 105.4°}$$

$$= 622N$$

$$T_2 = \frac{700 \times \sin 133.5°}{\sin 105.4°}$$

$$= 527\ N$$

<u>Question 15</u>

A particle P of mass 1.2 kg is acted upon by force $F_1 = (15\,N, 090°)$, $F_2 = (20\,N, 000°)$, $F_3 = (30\,N, 210°)$ and $F_4 = (k\,N, \alpha°)$. If the acceleration of the particle is

$\alpha = (14\,ms^{-2}, 240°)$, find, correct to the nearest whole number F_4.

Most candidates attempted this question. Candidates were given three forces and to find the fourth force to the nearest whole number given the acceleration.

Most candidates resolved the forces very well. However, an average number of candidates were able to proceed to find the fourth force (F_4) as demanded. Candidates were expected to solve the question as follows:

$$\begin{pmatrix}15sin90°\\15cos90°\end{pmatrix} + \begin{pmatrix}20sin0°\\20cos0°\end{pmatrix} + \begin{pmatrix}30sin210°\\30cos210°\end{pmatrix} + F_4 = 1.2\begin{pmatrix}14sin240°\\14cos240°\end{pmatrix}$$

$$\begin{pmatrix}15\\0\end{pmatrix} + \begin{pmatrix}0\\20\end{pmatrix} + \begin{pmatrix}-15\\-25.981\end{pmatrix} + F_4 = \begin{pmatrix}-14.5488\\-8.4\end{pmatrix}$$

$$\begin{pmatrix}0\\-5.981\end{pmatrix} + F_4 = \begin{pmatrix}-14.5488\\-8.4\end{pmatrix}$$

$$F_4 = \begin{pmatrix}-14.5488\\-8.4\end{pmatrix} - \begin{pmatrix}0\\-5.981\end{pmatrix}$$

$$F_4 = \begin{pmatrix}-14.5488\\-2.419\end{pmatrix}$$

$$k = \sqrt{(-14.5488)^2 + (-2.419)^2}$$

$$= 14.7485$$

$$tan\theta = \frac{-2.419}{-14.5488}$$

$$\theta = tan^{-1}\left(\frac{-2.419}{-14.5488}\right)$$

$$= 9.44°$$

$$\alpha = 270° - 9.44° \text{ (or } 180° + 80.56°)$$

$$= 260.56°$$

$$\therefore F_4 = (15N, 261°)$$

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$$\begin{pmatrix}15sin90°\\15cos90°\end{pmatrix} + \begin{pmatrix}20sin0°\\20cos0°\end{pmatrix} + \begin{pmatrix}30sin210°\\30cos210°\end{pmatrix} + \begin{pmatrix}ksin\alpha\\kcos\alpha\end{pmatrix} = 1.2\begin{pmatrix}14sin240°\\14cos240°\end{pmatrix}$$

$$\begin{pmatrix}15\\0\end{pmatrix} + \begin{pmatrix}0\\20\end{pmatrix} + \begin{pmatrix}-15\\-25.981\end{pmatrix} + \begin{pmatrix}ksin\alpha\\kcos\alpha\end{pmatrix} = \begin{pmatrix}-14.549\\-8.4\end{pmatrix}$$

$$\begin{pmatrix}0\\-5.981\end{pmatrix} + \begin{pmatrix}ksin\alpha\\kcos\alpha\end{pmatrix} = \begin{pmatrix}-14.5488\\-8.4\end{pmatrix}$$

$$ksin\alpha = -14.549 \quad \ldots\ldots\ldots\ldots(1)$$

$$k\cos\alpha = -2.42 \quad \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

$$k^2\sin^2\alpha + k^2\cos^2\alpha = (-14.549)^2 + (-2.42)^2$$

$$k^2(\sin^2\alpha + \cos^2\alpha) = 217.5264$$

$$k = \sqrt{217.5264}$$

$$= 14.75\ N$$

$$\tan\alpha = \frac{-14.549}{-2.42}$$

$$= 6.0198$$

$$\alpha = \tan^{-1}(6.0198)$$

$$= 80.56°$$

$$\text{Required angle} = 80.56° + 180°$$

$$= 260.56°$$

$$\therefore F_4 = (15N, 261°)$$